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Newton Method for Shape Reconstruction of a burried Scatterer

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Abstract

A Newton method for reconstruction of the shape of the perfectly electrical conducting (PEC) object located inside arbitrary shaped dielectric cylinder through the measured far-field scattering data is given. The scattered field is represented as a single-layer potential which leads to an ill-posed integral equation of the first kind that is solved via Tikhonov regularization. The presented Newton based method combines ideas of both the iterative and decomposition methods and inherits the advantages of each of them, such as getting good reconstructions and not requiring a forward solver at each step. The numerical results show that the method yields good reconstruction.

Key words: Inverse scattering, integral equations, ill-posed problems.

1. Introduction

Inverse scattering of waves is a fundamental principle of applications such as radar and sonar techniques, nondestructive evaluation, geophysical exploration and medical imaging. In principle, in these applications, the effects of scattering objects on the propagation of the waves are exploited to obtain some information about the unknown object. As opposed to classical techniques of imaging such as computerized tomography, which are based on the fact that X rays travel along straight lines, inverse scattering problems take into account that the propagation of acoustic, electromagnetic and elastic waves has to be modeled by a wave equation. This means that inverse scattering requires a nonlinear model, where as inverse tomography does linear.

The detection and identification of buried objects using electromagnetic waves are the areas of current importance for applications in remote sensing. The considered problem has practical applications such as detection of underground mines, pipes and cables. Most papers concerning the inverse scattering calculation are in free-space background. Colton and Monk [1-3] have carried out a series of acoustic wave inverse scattering calculations for two- and three-dimensional sound-soft impenetrable targets of several shapes. Ochs [4] has also discussed the limited-aperture problem of inverse scattering by applying the same method. Inverse obstacle reconstruction problem in free space was treated by Kirsch et al. [5], Jones and Mao [6], and Zinn [7] using different inversion techniques. Based on the Newton-Kantorovitch method, Roger [8], Kristensson and Vogel [9], Murch et al. [10], and Tobocman [11] have also solved two-dimensional inverse scattering problems of this type.

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In the literature, there are few examples related to reconstruction of object buried in the medium. Chammeloux et al. [12] have applied the technique of computer tomography to process the images of buried cylindrical inhomogeneities. This method can avoid the calculation of the Green's function, but they only obtained an approximate image of induced current, instead of the image of dielectric constant. Chiu and Kiang [13] solved buried inverse obstacle problem by Newton-Kantorovitch method. However, this method needs forward solver at each iteration step that needs high computational time especially for this problem due to the numerical evaluation of the Green's function of the medium.

In this study, we are interested in shape reconstruction of a perfectly electrical conducting (PEC) object located inside the arbitrary shaped dielectric medium from measurements of the far field pattern. Major advantage of the proposed method is that method does not need forward solver at each iteration step. The proposed Newton based method, which is sometimes called as Hybrid method, has been applied for shape reconstruction of the sound hard object or perfectly magnetic conductors (PMC) [14] and cracks [15] in free space.

2. Newton method for Burried Scatterer

The presented Newton iteration starts with an initial estimate Γ_0 of the boundary ∂D of the buried scatterer as depicted fig.1. The scattered field in the closed exterior of Γ_0 can be expressed by using single layer potential [16] as

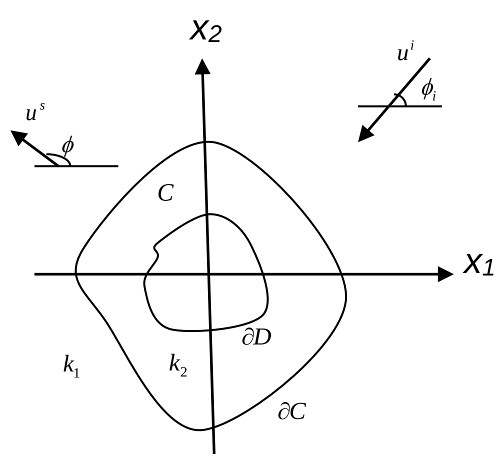

Figure 1. Geometry of the considered problem

$$(P_{\Gamma_0}\varphi)(x) = \int_{\Gamma_0} G(x,y)\varphi(y)ds(y), \qquad x \in R^2 / \Gamma_0 \tag{1}$$

with density $\varphi \in L^2(\Gamma_0)$ in the exterior of the surface Γ_0, where

$$G(x,y)=\begin{cases} G_t(x,y), & x\notin C,\ y\in C \\ \frac{i}{4}H_0^{(1)}(k_2|x-y|)+G_r(x,y), & x\in C, y\in C \end{cases} \tag{2}$$

represents the Green's function of Helmoltz equation in the domain containing dielectric cylinder (see Appendix A). Where $H_0^{(1)}(.)$ denotes the zero order Hankel function of the first kind. In (2), the terms $G_t(x,y)$ and $G_r(x,y)$ are smooth part of Green's function.

The far field pattern of the potential (1) for the scattered direction $\hat{x}=(\cos\phi,\sin\phi),\ \phi\in(0,2\pi)$ as shown in Fig.1, denoted by $(P_{\Gamma_0,\infty}\varphi)(\hat{x})$ can be derived from $G_t(x,y)$ while $|x|\to\infty$ as (see Appendix A)

$$(P_{\Gamma_0,\infty}\varphi)(\hat{x})=\frac{e^{i\pi/4}}{\sqrt{8\pi k_1}}\int_{\partial C}e^{-ik_1\hat{x}.g}\int_{\Gamma_0}\Phi(g,y)\varphi(y)ds(y)ds(g) \tag{3}$$

Because of the fact that the scattered field u^s is uniquely determined by its far field pattern u_∞, the density φ can be seen to be the unique solution of the ill posed integral equation

$$(P_{\Gamma_0,\infty}\varphi)(\hat{x})=u_\infty(\hat{x}) \tag{4}$$

Due to its analytic kernel, integral equation in (8) is severely ill posed [20]. However, the operator $P_{\infty,\Gamma_0}:L^2(\Gamma_0)\to L^2(\Omega)$ in (4) is known to be injective and has dense range. Therefore, Tikhonov regularization can be applied for a stable approximate solution of (4), that is, the ill-posed equation (8) is replaced by

$$\alpha\varphi+P^*_{\Gamma_0,\infty}P_{\Gamma_0,\infty}\varphi=P^*_{\Gamma_0,\infty}u_\infty \tag{5}$$

with some positive regularization parameter α and the adjoint $P^*_{\Gamma_0,\infty}$ of $P_{\Gamma_0,\infty}$.

For the further description of the reconstruction scheme we represent the curve Γ_0 by a regular parameterization

$$\Gamma_0=\{z_0(t):t\in[0,2\pi)\} \tag{6}$$

with a 2π periodic function $z_0 : R \to R^2$. Searching the location where the boundary condition (2) is satisfied, we approximate the total field u by the Taylor formula of order one with respect to the normal direction at Γ_0. For this purpose, we try to update in the form

$$\Gamma_1 = \{z_1(t) = z_0(t) + h(t)v_0(t) : t \in [0,2\pi)\} \tag{7}$$

where v_0 denotes the outward normal vector to Γ_0 and $h : IR \to IR$ is a sufficiently small 2π periodic function. The normal vector can be expressed through the parameterization (10) via

$$v_0(t) = \frac{(z_0'(t))^{\perp}}{|z_0'(t)|}, \quad t \in [0,2\pi) \tag{8}$$

where for any vector $a = (a_1, a_2)$, we set $a^{\perp} = (a_2, -a_1)$. Then the first order Taylor formula requires the update function h to satisfy

$$u + \left.\frac{\partial u}{\partial v_0}\right|_{\Gamma_0} h = 0 \tag{9}$$

Once the single layer density φ is known from (9), the values u and normal derivative $\partial u / \partial v_0$ of the total field on Γ_0 can be obtained through the jump relations for the single-layer potential [16], that is, by

$$u(x) = u^0(x) + \int_{\Gamma_0} \phi(x,y)\varphi(y)ds(y), \qquad x \in \Gamma_0 \tag{10}$$

$$\frac{\partial u}{\partial v}(x) = \frac{\partial u^0}{\partial v}(x) + \int_{\Gamma_0} \frac{\partial \phi(x,y)}{\partial v(x)}\varphi(y)ds(y) - \frac{1}{2}\varphi(x), \qquad x \in \Gamma_0 \tag{11}$$

where $u^0(x)$ is the total field in the absence of the PEC object.The integrals in (10) and (11) can be accurately evaluated by the quadrature rules as described in [16]

Since the solution of (9) is sensitive to errors in the normal derivative of u in the vicinity of zeros, Eq. (9) is solved in a stable way by the least squares method. For this purpose, we express h_0 in terms of the basis functions $\omega_1, \omega_2, ...\omega_J$ by

$$h(t) = \sum_{j=1}^{J} a_j \omega_j(t), \quad t \in [0,2\pi) \tag{12}$$

with possible choices of the basis functions given by splines or trigonometric polynomials. Then, we satisfy (9) in a penalized least squares sense, that is, the coefficients $a_1, a_2, ...a_J$ in (12) are chosen such that for a set of collocation points $t_1, t_2, ...t_N$ in $[0,2\pi)$ the penalized least squares sum

$$\sum_{n=1}^{N} \left| u(z_0(t_n)) + \frac{\partial u}{\partial v}(z_0(t_n)) \sum_{j=1}^{J} a_j \omega_j(t_n) \right|^2 + \beta \sum_{j=1}^{J} a_j^2 \quad (13)$$

with some regularization parameter $\beta > 0$ is minimized. After the solution of (13), updated boundary Γ_1 is obtained by using (7) and the same procedure applied for Γ_1.

with some regularization parameter $\beta_2 > 0$ is minimized. Then, finally $h(t)$ is inserted in (7) to obtain the updated boundary Γ_1. This procedure of alternatingly solving (5) and (9) now is iterated in an obvious fashion until some stopping criterium is satisfied.

3. Numerical Results

In the following examples, the degree of the trigonometric polynomial used for the approximation of the boundary is denoted by J. The Tikhonov regularization parameters for (9) is denoted by α, the number of Newton steps is denoted by T, the penalty factor in (13) by β, and the incidence angle by ϕ_0. In all examples, we used $N = 50$ collocation points. The wave number of background medium is chosen as $k_1 = 1$, and the iterations were started with circle whose center at the origin and radius 0.5m. In order to avoid an inverse crime, the synthetic data were obtained by solving the combined single- and double-layer boundary integral equation for the direct scattering problem by the Nyström method as described in [16] with 100 discritization points. Integral equations for obtaining total field and Green's function of domain containing dielectric cylinder are solved by Nyström method [16] with using 150 discritization points. As other parameters, we choose, $\alpha = 10^{-6}$, $\beta = 0.001, J = 4$ and $T = 7$.

We consider the reconstruction of the PEC object located inside a circular cylinder whose wave number and radius are $k_2 = 1.8$, $a = 2$ respectively. The angle of incidence is chosen as $\phi_0 = 0$. The parameterization of the boundaries of dielectric and PEC objects given respectively as

$$\partial C = \{2(\cos t, \sin t),\ t \in [0,2\pi)\}\ [m]$$

$$\partial D = \{(1 + 0.2\sin(3t))(\cos t, \sin t),\ t \in [0,2\pi)\}\ [m]$$

Dielectric cylinder, exact and reconstructed shapes of PEC are depicted in Fig.2.

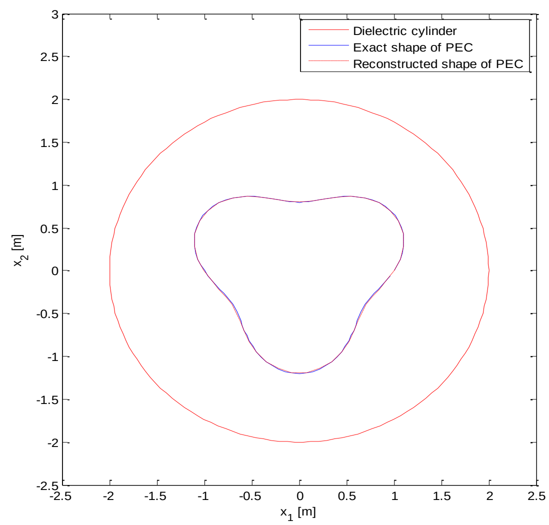

Figure 2. The reconstruction of the PEC object inside dielectric circular cylinder

Conclusions

In this study, the shape reconstruction of PEC object located inside a dielectric cylinder through the measured far-field scattering data is given. The scattered field is represented as a single-layer potential which leads to an ill-posed integral equation of the first kind that is solved via Tikhonov regularization. The field and its normal derivative are obtained by the single layer potential for an initial estimate. By applying the Newton method, a new estimate of boundary is obtained in the sense of least square. The main advantage of the proposed iterative method is that this method does not need forward solver for each iteration step that needs high computational time As seen from the figure obtained, the presented method gives good reconstruction.

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